Reconsidering Superconnexivity in Boolean Connexive Logics

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Connexive logics are logics where the following schemas are valid:

$$N(A > NA) \qquad \text{(Aristotle's Thesis)}$$
$$N(NA > A) \qquad \text{(Variant of Aristotle's Thesis)}$$
$$(A > B) > N(A > NB) \qquad \text{(Boethius' Thesis)}$$
$$(A > NB) > N(A > B) \qquad \text{(Variant of Boethius' Thesis)}$$

for a conditional $>$ and a negation N of a propositional language $\mathcal{L}$. There are several ways to strengthen these principles (see [1]). One of the most natural way to strengthen them is to require that [4]:

$$NA > A \text{ be unsatisfiable} \qquad \text{(Unsat1)}$$
$$A > NA \text{ be unsatisfiable} \qquad \text{(Unsat2)}$$
$$A > B \text{ and } A > NB \text{ not be simultaneously satisfiable.} \qquad \text{(Unsat3)}$$

Recently Omori and Kapsner have suggested in [5] to express such requirements in the object language using the following schemas:

$$(NA > A) > \bot \text{ and } (NA > A) > \bot \text{ are valid} \qquad \text{(Super-(Bot)-Aristotle)}$$
$$(A > B) > ((A > NB) > \bot) \text{ and } (A > NB) > ((A > B) > \bot) \text{ are valid} \qquad \text{(Super-(Bot)-Boethius)}$$

for some $\bot$ that expands the language $\mathcal{L}$, i.e., $\bot$ is not definable in $\mathcal{L}$.

In this paper we discuss three alternative ways to represent (Unsat1)–(Unsat3) in the object language without expanding it. We frame our discussion in the context of Relating Logics used to define Boolean Connexive Logics [3], [2], and we compare our approach to the one of Omori and Kapsner.

References

[1] L. Estrada-González and E. Ramírez-Cámara. A comparison of connexive logics. *IFCoLog Journal of Logics and their Applications*, 3:341–355, 2016.

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[4] A. Kapsner. Strong connexivity. *Thought: A Journal of Philosophy*, 1:141–145, 2012.

[5] A. Kapsner and H. Omori. Superconnexivity reconsidered. *Electronic Proceedings in Theoretical Computer Science*, 358:160–173, 2022.