Relating Semantics in Application to Boolean Connexive Logics Closed Under Negation and Demodalization

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In the presentation, we will introduce a specific classes of Boolean Connexive Logics (**BCL**) and Modal Boolean Connexive Logics (**MBCL**) that satifsy specific conditions in *relating semantics*. Mentioned conditions are following:

(i) being closed under a multiple negation,

(ii) being closed under demodalization.

Both families of logics are *connexive logics* defined by the means of *relating semantinc*, in which all Boolean connectives are interpreted in a standard way, except the implication, which is interpreted as a mono-relating connective.

A logic is *modal Boolean connexive* iff it is Boolean connexive and consists in its language modal operator(s) $\Box$ (or $\Diamond$) that behave(s) like modal operator(s). As a *connexive logic*, we understand any logic that contains Aristotle's and Boethius' Theses.

In the papers [1, 2, 6] Tomasz Jarmużek, Jacek Malinowski and Rafał Palczewski introduced the term Boolean Connexive Logics and defined both mentioned classes. This was done by the means of relating semantics, that allowed them to define connexive implication and to interpret all the other connectives, classically. Thus, **MBCL** and **BCL** contains all the classical laws, for we keep what is not contested in Classical Propositional Logic, while accepting the connexive laws for the relating implication.

Relating semantics was used as a framework for modelling BCL in [3] in a following way. Let For be a class of BCL formulas constructed in a standard way from variables $\mathbf{Var} = \{p, q, r, p_1, q_1, r_1 ...\}$, and connectives $\{\neg, \wedge, \vee, \rightarrow\}$, Moreover, let $R \subseteq \mathsf{For} \times \mathsf{For}$ and $v : \mathbf{Var} \longrightarrow \{0, 1\}$. The pair $\langle v, R\rangle$ we will call a model and a class of such models, we will denote by $\mathcal{M}$. Truth condition for a formulas from For are stated as follows:

- $\langle v, R\rangle \models A$ iff $v(A) = 1$, when $A \in \mathbf{Var}$;
- $\langle v, R\rangle \models \neg A$ iff $\langle v, R\rangle \not\models A$;
- $\langle v, R\rangle \models A \wedge B$ iff $\langle v, R\rangle \models A$ and $\langle v, R\rangle \models B$;
- $\langle v, R\rangle \models A \vee B$ iff $\langle v, R\rangle \models A$ or $\langle v, R\rangle \models B$;
- $\langle v, R\rangle \models A \rightarrow B$ iff $[\langle v, R\rangle \not\models A$ or $\langle v, R\rangle \models B]$ and $R(A, B)$;

The class $\mathcal{M}$ was narrowed down to adequately model Aristotle's and Boethius' theses by imposing listed conditions on the relation R for any $A, B \in \mathbf{Var}$:

(a1) $R(A, \neg A)$

(a2) $R(\neg A, A)$

(b1) $R(A, B) \Rightarrow \bar{R}(A, \neg B), R(A \to B, \neg(A \to \neg B))$

(b2) $R(A, B) \Rightarrow \bar{R}(A, \neg B), R(A \to \neg B, \neg(A \to B))$

For **MBCL**, which dictionary is extended in a standard way by addition of modal operator(s) $\Box$ (or $\Diamond$)), intepretation of formulas is accordingly changed and complicated. *Relating semantics* are extended with use of possible world semantics to the combined semantics that connect both approaches. The formal construction is based on the notion of a combined frame which is nothing more but relativiation of a *relating relation* to the considered possible world.

Most of the systems presented in [1, 2] were axiomatised in [5]. Here we would like to also present axiomatisation for the **BCL** and **MBCL** defined by the conditions of demodalisation and closure under multiple negations. The semantic condition of demodalisation have three basic forms:

$$R(A, B) \Rightarrow R(\mathsf{d}(A), \mathsf{d}(B)) \tag{1}$$

$$R(\mathsf{d}(A), \mathsf{d}(B)) \Rightarrow R(A, B) \tag{2}$$

$$R(A, B) \Leftrightarrow R(\mathsf{d}(A), \mathsf{d}(B)) \tag{3}$$

where $\mathsf{d}(A) = \begin{cases} A, \text{ if } A \text{ is a variable} \\ \neg\mathsf{d}(B), \text{ if } A = \neg B \\ \mathsf{d}(B) * \mathsf{d}(C), \text{ if } A = B * C \text{ and } * \in \{\wedge, \vee, \to\} \\ \mathsf{d}(B), \text{ if } A = *B \text{ and } * \in \{\Box, \Diamond\}. \end{cases}$

Whereas the condition of closure under negation, we presented in [1] is expressed as follows:

$$R(A, B) \Rightarrow R(\neg A, \neg B) \tag{4}$$

The logics determined by the condition were also axiomatised in [5]. In the paper we propose the generalisation of the condition by the means of *relating semantics*:

$$R(\underbrace{\neg \ldots \neg}_{k} A, \underbrace{\neg \ldots \neg}_{l} B) \Rightarrow R(\underbrace{\neg \ldots \neg}_{m} A, \underbrace{\neg \ldots \neg}_{n} B) \tag{5}$$

where k, l, m, n are numbers of occurrence of negation. For any k, l, m, n we present adequate axiomatic systems. All of the axiomatic systems in the paper are proved to be complete in the general way given in [4].

Literatura

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